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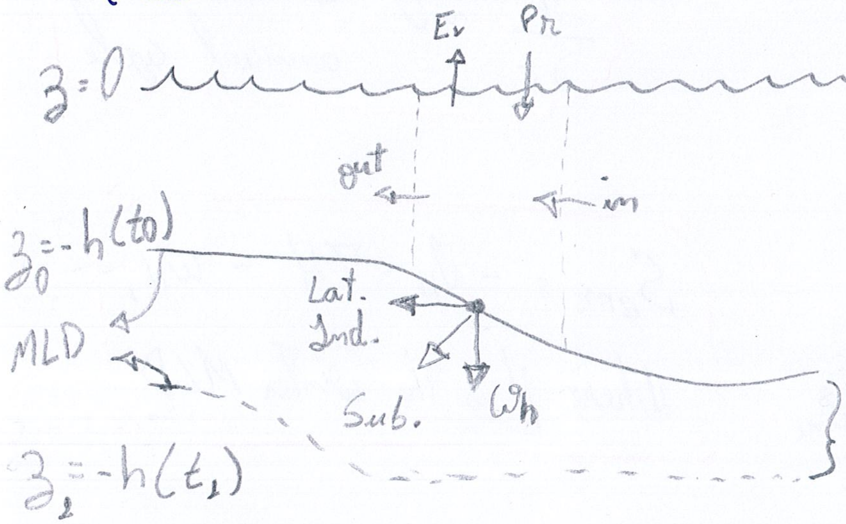
On Water Subduction in an Eddying Ocean

by Felipe Vilela-Silva

1 Analyze the volume fluxes within the mixed layer

(Cushman-Roisin, 1987)



$\frac{\partial h}{\partial t}$, rate of deepening

$$\boxed{\begin{gathered}\frac{Dh}{Dt} = Ev - Pr - S \\ \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = Ev - Pr - S\end{gathered}}$$

(i) $\frac{Dh}{Dt} >> Ev - Pr << S$, Evaporation minus precipitation is much lower than the other fluxes

"$O(Ev - Pr) \sim 10^0 \text{ m year}^{-1}$ and $O(S) \sim 10^2 \text{ m year}^{-1}$"

So, $-S = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} + h\frac{\partial v}{\partial y}$

$$S = -1 * \left(\frac{\partial h}{\partial t} + \underbrace{\vec{u}\cdot\nabla h}_{\text{Lateral Induction}} + \underbrace{h\nabla\cdot\vec{u}}_{\text{Vertical Vel. at MLD}}\right)$$

Vertical Vel. at MLD: $\frac{\partial w}{\partial z} \overset{\text{continuity}}{=} -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) =$

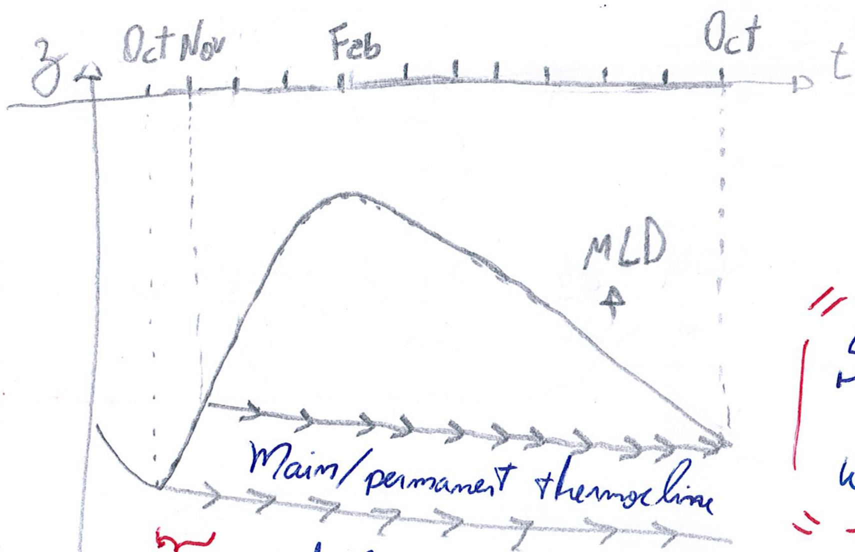
$$= \frac{w(z=0) - w(z=h)}{0 - (-h)} = -\nabla\cdot\vec{u} \Rightarrow w_h = h\nabla\cdot\vec{u}$$

② She annual subduction rate

* Considerations [from (i) to (iv)]

(i) Fluid leaving the mixed layer when it is deep during winter and spring that irreversebly enters the permanent thermocline

(ii) Fluid subducted from summer mixed layer is generally re-intrained as the mixed layer depens the following winter.



$$\frac{\partial h}{\partial t} \sim 0 \quad \text{within the annual cycle}$$

$$S_{ann} = -\vec{v}_H \cdot \nabla H - w_H,$$

where H is the winter MLD

the subduction that occured within this short period is known as the effective subduction because this water parcels won't change information with the atmosphere within the annual cycle. (Stommel, 1979).

(iii) She process above is also known as "mixed layer demon" (Stommel, 1979)

↳ She Mixed Layer Demons are an analogy to James Maxwell studies of kinematic theory of gases. She Maxwell's demons were in charge of estimating the velocity of molecules moving from two sides of a gas chamber divided by trap doors. She demons were also fast enough to open and close the trap doors in time to allow the fast molecules to acumulate in one side of the gas chamber.

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(iv) The instantaneous and annual subduction represent kinematic estimates and do not consider tracer advection.

→ Tracer experiments confirm that fluid is only transferred from the mixed layer into the main thermocline between late winter and early spring.

→ However, eddy stirring enhances the rate at which the tracer spreads into unventilated regions (Williams et al., 1995).

3 Subduction of water masses in an eddying ocean (David Marshall, 1977)

(i) Consider a two-dimensional ocean (y, z). Following Cushman-Roisin (1987)

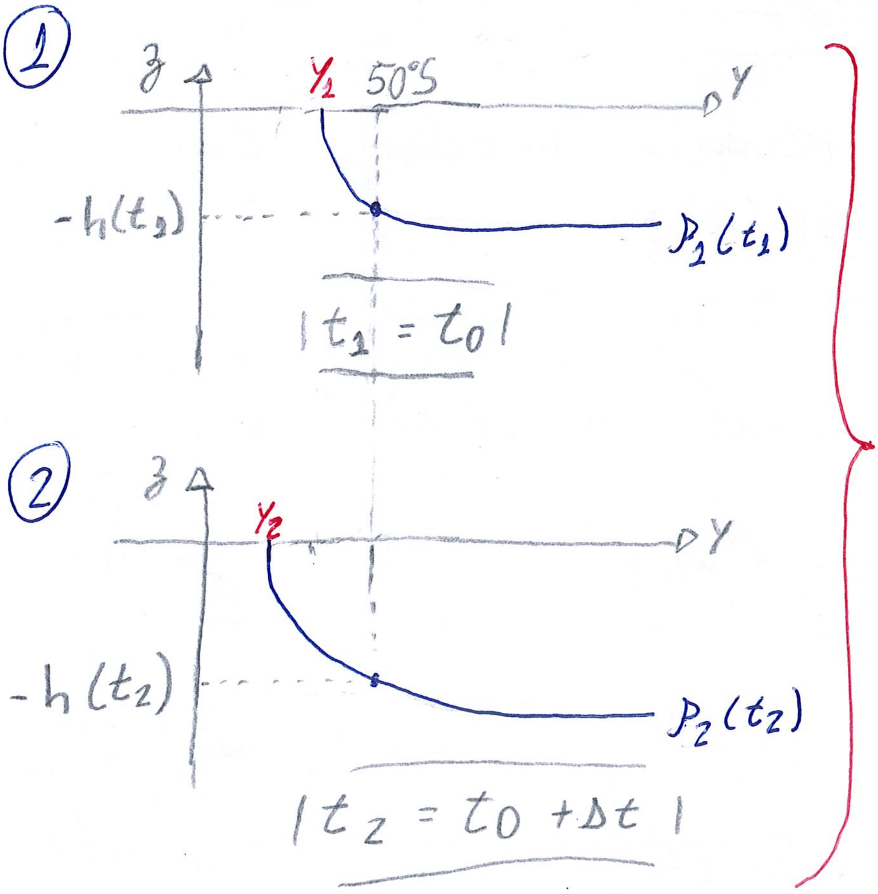
$$S = -\left(\frac{\partial h}{\partial t} + v_b \frac{\partial h}{\partial y} + w_b\right) \qquad (1) \qquad \left.\right|_{z=-h}$$

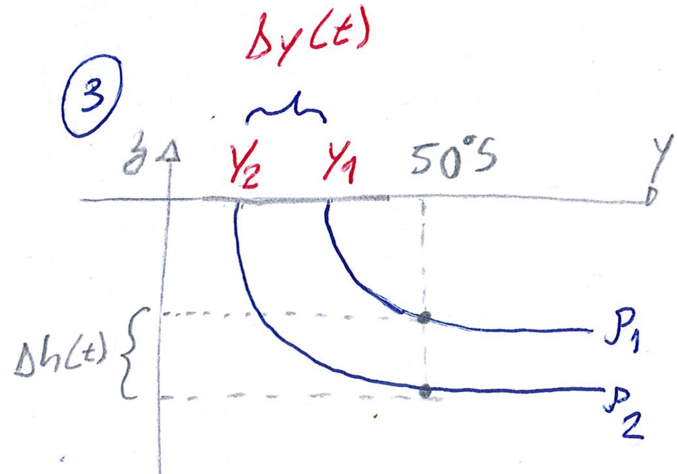
→ One can estimate climatological subduction by substituting Eulerian-mean velocities into (1) over regions where subduction of water masses is dominated by Eulerian-mean advection (e.g., subtropical gyres).

→ However, (1) is not suitable in regions of intense baroclinic instability (e.g., Southern Ocean) because the contribution of tracer advection from eddies is of the same order of large-scale subduction due to Eulerian-mean advection.

→ How to quantify the eddy-induced subduction of a water mass?

↳ A simple Eulerian time-average of Eq. (2) is not appropriate / the terms in (2) are not non-correlated in time because <u>MLD</u> and <u>position</u> of the outcrop isopycnal <u>change</u> at each time step. So, a Reynolds decomposition is not valid in (2). In other words, it's not right to obtain S' as $S(t) - \bar{S}$





↳ It's possible to obtain the <u>time-mean subduction rate</u> in a Lagrangian frame of reference which follows the time-evolving surface density outcrops, and taking into account the outcrop area (Δy) as illustrated above//

⇒ So, the <u>net subduction</u> of the water mass within a density range ($\rho_1 \leq \rho \leq \rho_2$) is given by

$$\boxed{S(t)\,\Delta y(t) = -\left(\frac{\partial h}{\partial t} + w_b \frac{\partial h}{\partial y} + w_b\right) \cdot \Delta y}$$

$$S(t)\,\Delta y(t) = -\left(\frac{\partial h}{\partial t}\Delta y + v_b \frac{\partial h}{\partial y}\Delta y + w_b\,\Delta y\right)$$

- Now, we can apply the Reynolds decomposition ($v = \bar{v} + v'$, $\Delta y = \overline{\Delta y} + \Delta y'$, etc).

$$= -\left[\frac{\partial(\bar{h} + h')}{\partial t}\left(\overline{\Delta y} + \Delta y'\right) + \right.$$

$$+ \left(\bar{v}_b + v_b'\right)\frac{\partial(\bar{h} + h')}{\partial y}\left(\overline{\Delta y} + \Delta y'\right) +$$

$$\left. + \left(\bar{w}_b + w_b'\right)\left(\overline{\Delta y} + \Delta y'\right)\right]$$

$$\overline{S(t)\,\Delta y(t)} = -\left[\frac{\partial \bar{h}}{\partial t}\overline{\Delta y} + \frac{\partial \bar{h}}{\partial t}\overline{\Delta y'} + \cancelto{0}{\frac{\overline{\partial(h')}}{\partial t}\overline{\Delta y}} + \overline{\frac{\partial(h')}{\partial t}\Delta y'} + \right.$$

$$+ \bar{v}_b \frac{\partial \bar{h}}{\partial y}\left(\overline{\Delta y} + \Delta y'\right) + \cancelto{0}{\bar{v}_b \frac{\overline{\partial(h')}}{\partial y}\left(\overline{\Delta y} + \Delta y'\right)} +$$

$$+ \cancelto{0}{\overline{v_b'}\frac{\partial \bar{h}}{\partial y}\left(\overline{\Delta y} + \Delta y'\right)} + \overline{v_b'\frac{\partial(h')}{\partial y}\left(\overline{\Delta y} + \Delta y'\right)} +$$

$$\left. + \overline{w_b}\,\overline{\Delta y} + \overline{w_b}\,\overline{\Delta y'} + \cancelto{0}{\overline{w_b'}\,\overline{\Delta y}} + \overline{w_b'\,\Delta y'}\right]$$

↳ Next page,,

$$= -\Big[\overline{\frac{\partial \bar h}{\partial t}}\,\overline{\Delta y} + \overline{\frac{\partial h'}{\partial t}\Delta y'} + \bar v_b \overline{\frac{\partial \bar h}{\partial y}}\,\overline{\Delta y} +$$

$$+ \cancelto{0}{\bar v_b \overline{\frac{\partial \bar h}{\partial y}}\,\overline{\Delta y'}} + \overline{v_b' \frac{\partial h'}{\partial y}}\,\overline{\Delta y} + \overline{v_b' \frac{\partial h'}{\partial y}\Delta y'}$$

$$+ \overline{w_b}\,\overline{\Delta y} + \overline{w_b' \Delta y'}\Big]$$

$$\overline{S(t)\Delta y(t)} = -\Big[\frac{\partial \bar h}{\partial y}\overline{\Delta y} + \bar v_b \frac{\partial \bar h}{\partial y}\overline{\Delta y} + \bar w_b \overline{\Delta y}\Big]$$

$$-\Big[\overline{\frac{\partial h'}{\partial t}\Delta y'} + \overline{v_b' \frac{\partial h'}{\partial y}\Delta y'} + \overline{w_b' \Delta y'}\Big]$$

$$- \overline{v_b' \frac{\partial h'}{\partial y}}\,\overline{\Delta y}$$

→ Finally, dividing by the mean outcrop spacing (i.e., $\overline{\Delta y}$) returns the "Waterman subduction rate per unit surface area",

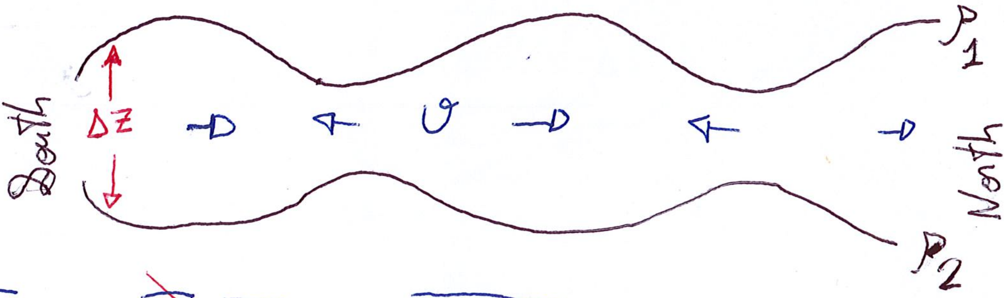
$$\bar S^{WM} = \frac{\overline{S\Delta y}}{\overline{\Delta y}} = \underbrace{-\left(\frac{\partial \bar h}{\partial t} + \bar v_b \frac{\partial \bar h}{\partial y} + \bar w_b\right)}_{S_{Eulerian}}$$

$$\underbrace{-\frac{1}{\overline{\Delta y}}\overline{\left(\frac{\partial h}{\partial t} + v_b \frac{\partial h}{\partial y} + w_b\right)'\Delta y'} - \overline{v_b' \frac{\partial h'}{\partial y}}}_{S_{Eddy}\text{, or eddy-induced subduction}} \qquad (2)$$

↳ Eq. (2) contains both the terms to estimate the transfer rate between mixed layer and thermocline due to Eulerian-mean advection and eddies.

↳ However, Eq. (2) is not in a form that would be simple to estimate eddy subduction rates from observations or to parameterize subduction in a numerical model.
(I understood that it's not simple to compute Δy in observations.)

a) Bolus Velocity



$$T = v\,\Delta Z$$

$$\overline{T} = \overline{v}\,\overline{\Delta Z} + \cancelto{0}{\overline{v}\,\overline{\Delta Z'}} + \cancelto{0}{\overline{v'}\,\overline{\Delta Z}} + \overline{v'\Delta Z'}$$

$$\overline{T} = \overline{v}\,\overline{\Delta Z} + \overline{v'\Delta Z'} \rightarrow \text{bolus transport}$$

* The correlation between fluid velocity and the perturbation layer thickness means that a greater volume is transported north when $v>0$, than is transported south when $v<0$.

→ Eddies not only diffuse tracers along isopycnals, but also provide an additional advection by the "bolus velocity".

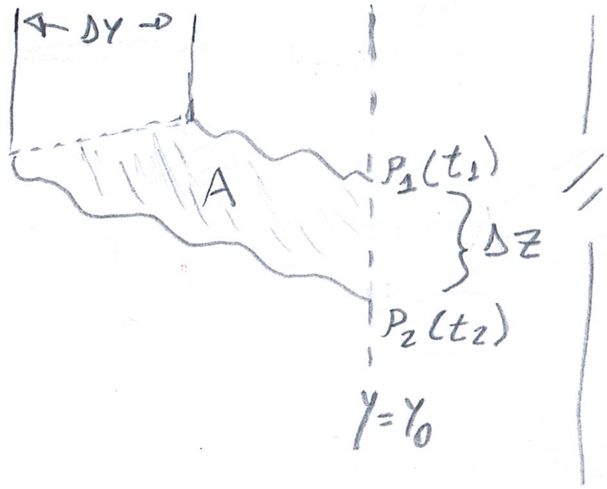
$$v^* = \frac{\overline{T}}{\overline{\Delta Z}} = \frac{\overline{v'\Delta Z'}}{\overline{\Delta Z}} \qquad (3)$$

→ We obtain the vertical bolus velocity (w^*) by applying the continuity condition in (3),

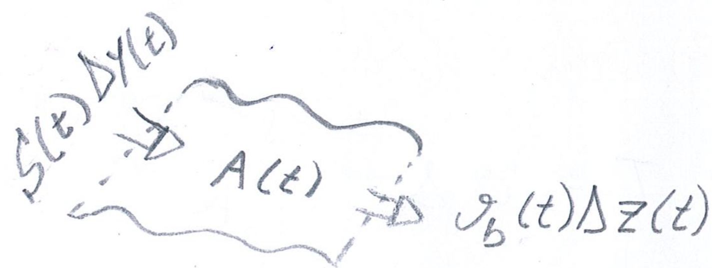
$$\frac{\partial v^*}{\partial y} + \frac{\partial w^*}{\partial z} = 0 \Rightarrow w^*(z=-h) = \bar{h}\,\frac{\partial v^*}{\partial y}, \quad w^*(z=0) = 0$$

b) Eddy subduction using the bolus velocity

(i) First, consider the Area A below



→ What are the fluxes in the Area A?



(ii) The area A can change only through subduction from the surface mixed layer or a lateral transport across $Y=Y_0$,

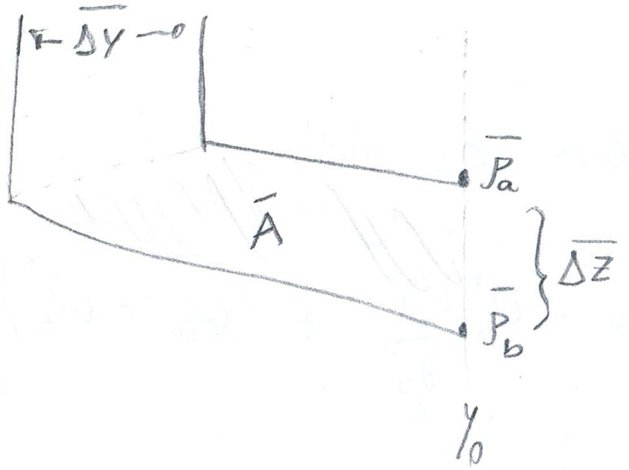
$$\frac{DA}{Dt} = S\,\Delta Y - v_b\,\Delta Z.$$

(iii) Averaging over several eddy life-cycles gives

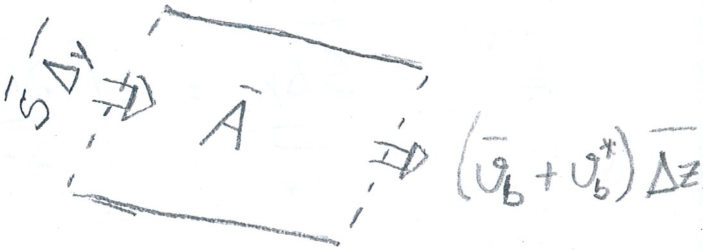
$$\overline{\frac{DA}{Dt}} = \overline{S\,\Delta Y} - (\overline{v_b} + v_b^*)\,\overline{\Delta Z} \qquad (4)$$

↳ here, it contains both the Eulerian and eddy-induced terms from Eq. (2) //

(iv) Now, let's consider an analogous problem for the Area $\bar{A}$ contained between the time-mean isopycnals $\bar{\rho}_a$ and $\bar{\rho}_b$, the mixed layer, and Y_0 below.



* What are the fluxes in the Area $\bar{A}$?



$\Delta\rho = \bar{\rho}_b - \bar{\rho}_a$

$0 < \Delta\rho < 10^{-2}\ \mathrm{Kg\ m^{-3}}$

In this configuration, we find,

$$\frac{D\bar{A}}{Dt} = \bar{S}\,\overline{\Delta y} - (\bar{v}_b + v_b^*)\,\overline{\Delta z}.$$

Note that

$$\overline{S\,\Delta y} \neq \bar{S}\,\overline{\Delta y}$$

* Here, $\bar{S}\,\overline{\Delta y} = -\left[\frac{\partial \bar{h}}{\partial t} + (\bar{v}_b + v_b^*)\frac{\partial \bar{h}}{\partial y} + (\bar{w}_b + w_b^*)\right]\overline{\Delta y}$

because we didn't average over several eddy life-cycles as in step (iii). So,

$$\frac{D\bar{A}}{Dt} = -\left[\frac{\partial \bar{h}}{\partial t} + (\bar{v}_b + v_b^*)\frac{\partial \bar{h}}{\partial y} + (\bar{w}_b + w_b^*)\right]\overline{\Delta y} - (\bar{v}_b + v_b^*)\,\overline{\Delta z} \quad (5)$$

(9) Finally, equating (4) and (5): $\boxed{\overline{\frac{DA}{Dt}} = \frac{D\bar{A}}{Dt}}$

$$\overline{S\Delta y} - (\bar{v}_b + v^*)\overline{\Delta z} = -\left[\frac{\partial \bar{h}}{\partial t} + (\bar{v}_b + v_b^*)\frac{\partial \bar{h}}{\partial y} + (\bar{w}_b + w_b^*)\right]\overline{\Delta y}$$

$$- (\bar{v}_b + v_b^*)\overline{\Delta z}$$

and using the arguments from Eq. (2),

$$\boxed{\bar{S}^{WM} = \frac{\overline{S\Delta y}}{\overline{\Delta y}} = -\left[\frac{\partial \bar{h}}{\partial t} + (\bar{v}_b + v_b^*)\frac{\partial \bar{h}}{\partial y} + (\bar{w}_b + w_b^*)\right]}$$